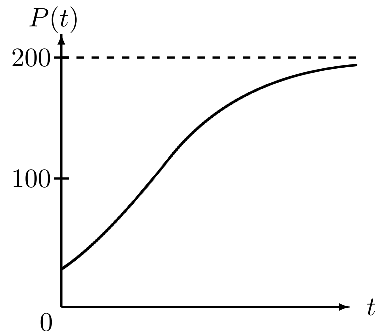
CalcBC Test 1 Name: Block: Seat:

70 minutes. Multiple choice should take about 2-3 minutes to solve, and worth about $\frac{20}{7} \approx 3$ points each. Free response questions should take about 15 each and is worth 50% of the exam.

1. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?
   (a) 3
   (b) 5
   (c) 6
   (d) 10
   (e) 12

2. The length of a curve from $x = 1$ to $x = 4$ is given by
$$\int_1^4 \sqrt{1 + 9x^4}dx$$
   If the curve contains the point $(1, 6)$ which of the following could be an equation for this curve?
   (a) $y = 3 + 3x^2$
   (b) $y = 5 + x^3$
   (c) $y = 6 + x^3$
   (d) $y = 6 - x^3$
   (e) $y = \dfrac{16}{5} + x + \dfrac{9}{5}x^5$

3. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\dfrac{dM}{dt} = 0.6M\left(1 - \dfrac{M}{100}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{x\to\infty} M(t)$?
   (a) 50
   (b) 100
   (c) 200
   (d) 500
   (e) 1000



4. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?
   (a) $\dfrac{dP}{dt} = 0.2P - 0.001P^2$
   (b) $\dfrac{dP}{dt} = 0.1P - 0.001P^2$
   (c) $\dfrac{dP}{dt} = 0.2P^2 - 0.001P^2$
   (d) $\dfrac{dP}{dt} = 0.1P^2 - 0.001P^2$
   (e) $\dfrac{dP}{dt} = 0.1P^2 + 0.001P^2$

5. The solution to the differential equation $\dfrac{dy}{dx} = 10xy$ with the initial condition $y(0) = 2$ is
   (a) $\ln(5x^2 + 2)$
   (b) $2\ln(5x^2)$
   (c) $e^{5x^2}+2$
   (d) $e^{5x^2}+1$
   (e) $2e^{5x^2}$

6. The table gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

x	$g'(x)$
-1.0	2
-0.5	4
0.0	3
0.5	1
1.0	0
1.5	-3
2.0	-6

(a) -6.5
(b) -1.5
(c) 1.5
(d) 2.5
(e) 3

7. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

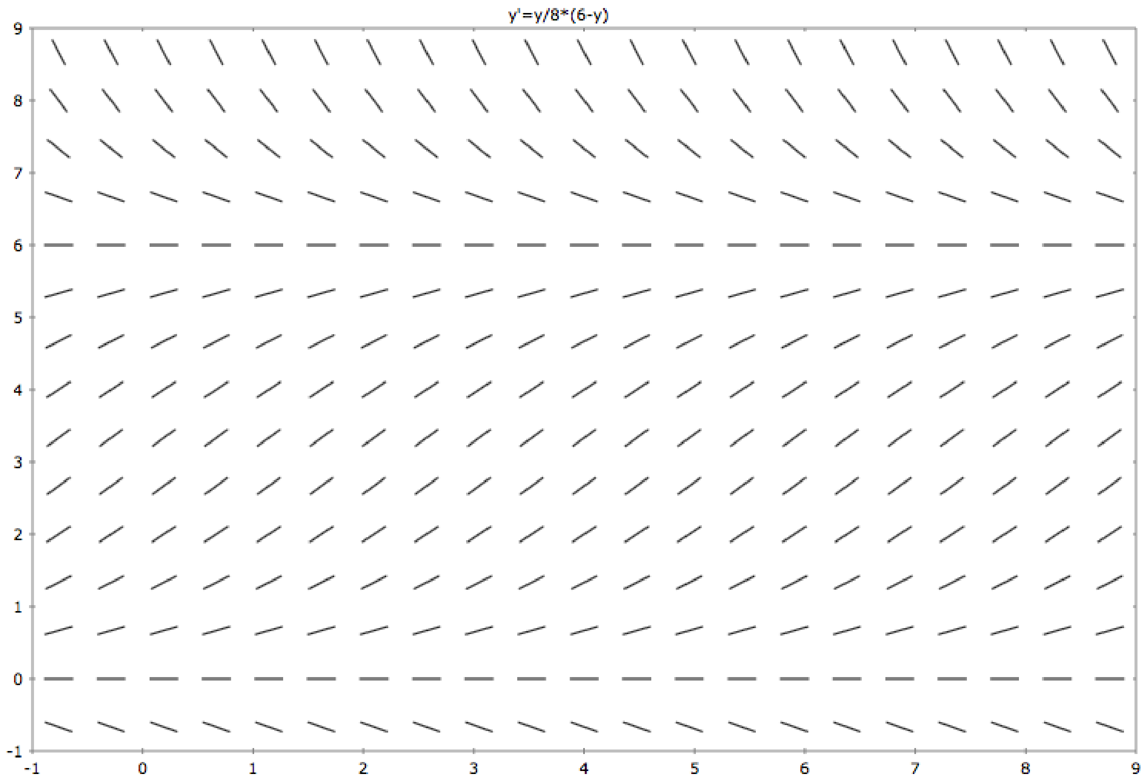
(a) $\int_a^b \sqrt{x + \cos^2(\sqrt{x})}dx$
(b) $\int_a^b \sqrt{1 + \cos^2(\sqrt{x})}dx$
(c) $\int_a^b \sqrt{\sin^2(\sqrt{x}) + \frac{1}{4x}\cos^2(\sqrt{x})}dx$
(d) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2(\sqrt{x})}dx$
(e) $\int_a^b \sqrt{\dfrac{1 + \cos^2(\sqrt{x})}{4x}}dx$

8. Consider the logistic differential equation

$$\frac{dy}{dt} = \frac{y}{8}(6 - y)$$

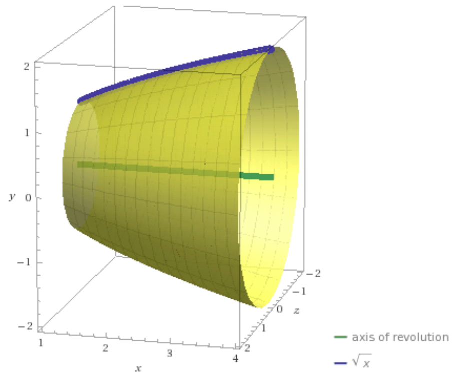
Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = 8$

(a) (2 points) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.



(b) (2 points) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) (1 point) Let $g = f(x)$ be the particular solution to the differential equation with $g(0) = 1$. About what time does the rate of g reach the fastest?



9. Let $f(x) = \sqrt{x}$ so that $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$

   (a) (2 points) Using your calculator, approximate the length of $f(x)$ from $x = 1$ to $x = 4$

   (b) (2 points) Using your calculator, approximate the area of the surface generated by revolving the curve about the x-axis for $1 \leq x \leq 4$
   Hint: The distance of the graph to the x-axis at any point x is $f(x)$.

   (c) (2 points) Find the exact value.
   Hint: After you simplify the integrand a bit (common denominator, etc), use u substitution for $\int \sqrt{u}\, du$ and remember to change the limits from values in terms of x to values in terms of u.

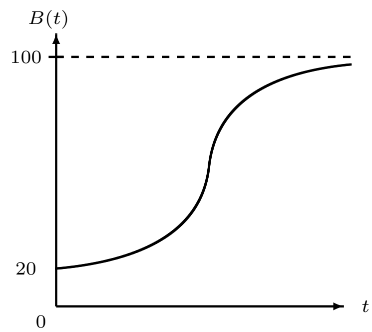
10. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{4}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with the initial condition $B(0) = 20$

(a) (2 points) Is the bird gaining weight faster when it weighs 30 grams or when it weighs 50 grams? Explain your reasoning.

(b) (2 points) Find $\frac{d^2B}{dt^2}$ in terms of of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the graph:



(c) (5 points) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.